DIRECT AND INVERSE ENERGY CASCADES IN A FORCED ROTATING TURBULENCE EXPERIMENT

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Abstract
We present experimental evidence for a double cascade of kinetic energy in a statistically stationary rotating turbulence experiment. Turbulence is generated by a set of vertical flaps which continuously injects velocity fluctuations towards the center of a rotating water tank. The energy transfers are evaluated from two-point third-order three-component velocity structure functions, which we measure using stereoscopic particle image velocimetry in the rotating frame. Without global rotation, the energy is transferred from large to small scales, as in classical three-dimensional turbulence. For nonzero rotation rates, the horizontal kinetic energy presents a double cascade: a direct cascade at small horizontal scales and an inverse cascade at large horizontal scales. By contrast, the vertical kinetic energy is always transferred from large to small horizontal scales, a behavior reminiscent of the dynamics of a passive scalar in two-dimensional turbulence. At the largest rotation rate the flow is nearly two-dimensional, and a pure inverse energy cascade is found for the horizontal energy. To describe the scale-by-scale energy budget, we consider a generalization of the Kármán-Howarth-Monin equation to inhomogeneous turbulent flows, in which the energy input is explicitly described as the advection of turbulent energy from the flaps through the surface of the control volume where the measurements are performed.

INTRODUCTION

Global rotation is a key ingredient of many geophysical and astrophysical flows. Through the action of the Coriolis force, rotating turbulence tends to approach two-dimensionality, i.e. invariance along the rotation axis [1]. Energetic 2D and 3D flow features therefore coexist in rotating turbulence, and the question of the direction of the energy cascade naturally arises: In 3D, energy is transferred from large to small scales [2] whereas it is transferred from small to large scales in 2D [3]. In rotating turbulence, energy transfers depend on the Rossby number Ro, which compares the rotation period Ω^{-1} to the turbulent turnover time. In the limit of vanishing Ro, 3D energy transfers occur through resonant triadic interactions [4] of inertial waves [5], which drive energy in an anisotropic *direct* cascade, with a net transfer towards slow, small-scale, nearly 2D modes [6, 7]. Exactly resonant triads cannot however drive energy from 3D modes to the exactly 2D mode: this decoupling implies that, if energy is supplied to the 3D modes only, the 2D mode should not be excited and no inverse cascade should be observed. In contrast, at moderate Rossby numbers, near-resonant triadic interactions, which are increasingly important as Ro is increased, allow for non-vanishing energy transfers between 3D and 2D modes [8, 9], thus providing a mechanism for the emergence of an inverse energy cascade in the 2D mode. This intermediate Rossby number regime is of first practical interest: the Rossby number of most laboratory experiments and geophysical/astrophysical flows is indeed of the order of $10^{-1} - 10^{-2}$. In these situations, a natural question is to what extent direct and inverse cascades may coexist, and what sets their relative amplitudes as the Rossby number is varied.

Although numerical simulations have provided particularly valuable insight about the conditions under which an inverse cascade takes place in rotating turbulence, the most common assumptions of homogeneity and narrow-band spectral forcing are of limited practical interest. In most flows encountered in the laboratory and in geophysical/astrophysical contexts, energy injection in a given control volume is indeed broadband and results from the spatial gradients of turbulent energy. As a consequence, the well-separated inverse and direct cascades observed in numerical simulations with a separating wave number fixed at the forcing wave number k_f are not relevant to describe real flows with boundary forcing.

RESULTS

Turbulence is generated by a set of vertical flaps which continuously inject velocity fluctuations towards the center of a rotating water tank (Fig. 1(a)) [10, 11, 12]. The flaps are vertically invariant, but instabilities in their vicinity induce 3D turbulent fluctuations, so the forcing injects energy both in the 2D and 3D modes. We compute the energy transfers in the plane normal to the rotation axis as the divergence $\Pi_\perp = \frac{1}{4}\boldsymbol{\nabla}_\perp \cdot \langle \overline{(\delta\mathbf{u}')^2 \delta\mathbf{u'}_\perp} \rangle$ of the two-point third-order velocity structure functions extracted from stereoscopic particle image velocimetry measurements in the rotating frame (the index$_\perp$ denotes the projection on the plane normal to the rotation). We provide an experimental evidence of a double energy cascade, direct at small scales and inverse at large scales (Fig. 1(b)). Since turbulence is statistically steady, the inverse cascade does not manifest through a temporal growth of kinetic energy, but it is characterized by a change of sign of the scale-by-scale energy flux. As the rotation rate is increased, the inverse cascade becomes more pronounced and spreads down to the smallest scales. As compared to previous experimental observations of an inverse cascade in rotating turbulence, here we provide for the first time a direct scale-by-scale measurement of the energy transfers in the horizontal plane. This allows us to distinguish between the horizontal transfers of vertical and horizontal kinetic energy. At the

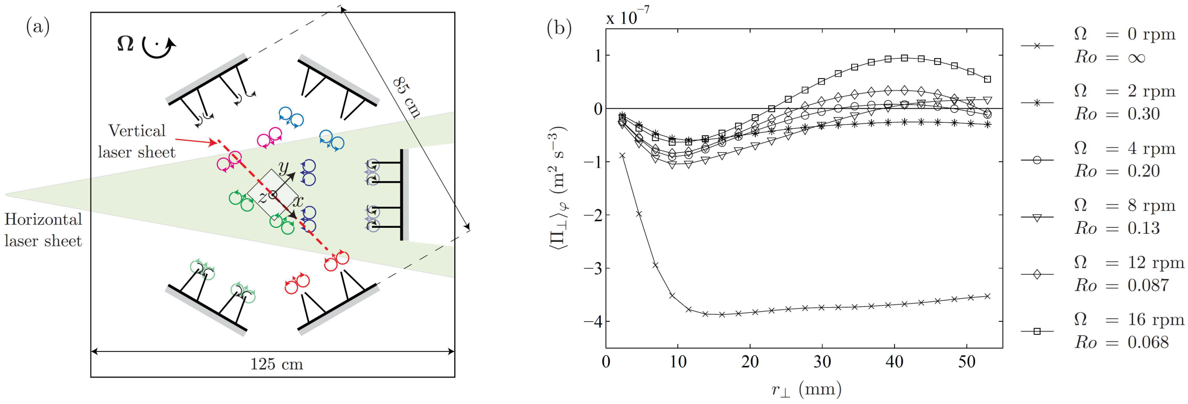

Figure 1. (a) Schematic of the experiment: an arena of 10 pairs of vertical flaps is placed in a parallelepipedic water tank rotating at angular velocity Ω. The rotation vector $\mathbf{\Omega}$ is vertical and the system is viewed from above. The rectangle at the center of the arena indicates the horizontal region where 2D-3C velocity fields are measured by stereoscopic particle image velocimetry. The drawing shows idealized vortex dipoles emitted by the generators, before they interact in the center of the arena. (b) Azimuthal average of the horizontal energy flux, $\langle \Pi_\perp \rangle_\varphi$, as a function of horizontal separation $r_\perp$ and for various rotation rates Ω. A negative value (resp. positive) corresponds to a direct (resp. inverse) energy transfer.

largest rotation rate, this double cascade of the total energy can be described as the superposition of an inverse cascade of horizontal energy and a direct cascade of vertical energy. This is consistent with the 2D3C dynamics expected in the limit of small (but finite) Rossby numbers, with the vertical velocity behaving as a passive scalar transported by the horizontal flow.

Contrary to numerical simulations, in which energy is usually supplied by a homogeneous body force acting on a prescribed narrow range of wave numbers, in most experiments and in many natural flows, energy is supplied at the boundaries. For a control domain away from those boundaries, energy is advected from the boundaries into the domain. This spatial flux of energy, which is strongly related to the inhomogeneities of the turbulent statistics, results in an effective broad-band energy injection term. In order to interpret the energy transfers in such an experiment, it is therefore necessary to separate the contributions from the spatial transport and from the scale-by-scale transfers. We have performed this analysis using the inhomogeneous generalization of the Kármán-Howarth-Monin equation, and we have measured directly the energy transport term for scales at which the velocity-pressure correlations can be neglected (quasi-homogeneous approximation). Because of this effective broad-band forcing, the inversion scale, which separates the direct and inverse cascades, is not directly prescribed by the geometry of the forcing device and decreases with the imposed rotation rate. Modelling this inversion scale as a function of the Rossby number and forcing geometry remains an open issue of first interest for flows of oceanic and atmospheric relevance, such as convectively-driven rotating flows.

This talk is based on the work presented in detail in reference [13].

References

[1] P.A. Davidson. *Turbulence in rotating, stratified and electrically conducting fluids.* (Cambridge University Press, Cambridge, 2013).
[2] U. Frisch. *Turbulence - The Legacy of A.N. Kolmogorov.* (Cambridge University Press, Cambridge, 1995).
[3] P. Tabeling. Two-dimensional turbulence: a physicist approach. *Physics Report* **362**: 1–62, 2002.
[4] C. Cambon and L. Jacquin. Spectral approach to non-isotropic turbulence subjected to rotation. *Journal of Fluid Mechanics* **202**: 295–317, 1989.
[5] H. Greenspan. *The theory of rotating fluids.* (Cambridge University Press, 1968).
[6] S. Galtier. Weak inertial-wave turbulence theory. *Physical Review E* **68**: 015301, 2003.
[7] C. Cambon, R. Rubinstein, and F. S. Godeferd. Advances in wave turbulence: rapidly rotating flows. *New Journal of Physics* **6**: 73, 2004.
[8] L.M. Smith and Y. Lee. On near resonances and symmetry breaking in forced rotating flows at moderate Rossby number. *Journal of Fluid Mechanics* **535**: 111–142, 2005.
[9] L. Bourouiba and P. Bartello. The intermediate Rossby number range and two-dimensional three-dimensional transfers in rotating decaying homogeneous turbulence. *Journal of Fluid Mechanics* **587**: 139–161, 2007.
[10] P. Billant and J.-M. Chomaz. Experimental evidence for a new instability of a vertical columnar vortex pair in a strongly stratified fluid. *Journal of Fluid Mechanics* **418**: 167–188, 2000.
[11] P. Augier, P. Billant, M.E. Negretti, and J.-M. Chomaz. Experimental study of stratified turbulence forced with columnar dipoles. *Physics of Fluids* **26**: 046603, 2014.
[12] B. Gallet, A. Campagne, P.-P. Cortet, and F. Moisy. Scale-dependent cyclone-anticyclone asymmetry in a forced rotating turbulence experiment. *Physics of Fluids* **26**: 035108, 2014.
[13] A. Campagne, B. Gallet, F. Moisy, and P.-P. Cortet. Direct and inverse energy cascades in a forced rotating turbulence experiment. *Physics of Fluids* **26**: 125112, 2014.