FULLY TURBULENT MEAN VELOCITY PROFILE FOR PURELY VISCOUS NON-NEWTONIAN FLUIDS

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Abstract The characteristic near wall behavior of turbulent flow of purely-viscous non-Newtonian fluids is discussed for both power-law (P.-L.) and Herschel-Bulkley (H.-B.) rheological models. A proper scaling is presented for H.-B. fluids to establish an analogy with power-law fluids with same flow index. To provide reference data for turbulent flow of non-Newtonian fluids, DNS simulations of power-law fluids are conducted in a rectangular channel for a large range of power-law indices (n = 0.5, 0.69, 0.75, 0.9, 1, 1.2). The DNS data show that the mean velocity profile in the viscous and logarithmic layers follow expressions of the form $u^+ = y^+$ and $u^+ = 2.5\,log(y^+) + B_n$ respectively, where B shows a logarithmic dependency on the flow index.Comparison with some experimental data shows the above formulation to be valid for Reynolds numbers (based on shear velocity) as high as 1000 .

INTRODUCTION

The basic equations that govern the flow of an incompressible fluid are the Navier-Stokes equations

$$\nabla.\mathbf{u} = 0; \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla.\tau \tag{1}$$

Supplemented by a proper rheological model, Eqs. (1) express the behavior of non-Newtonian fluids.
In the current work, our focus is restricted to power-law (P.-L.) and Herschel-Bulkley (H.-B.) fluids, so that the shear stress is modeled by

$$\tau = \tau_y + K\dot{\gamma}^n. \tag{2}$$

Here τ_y stands for the yield stress of a fluid, so that it reduces to zero for a P.-L. model.
It is obvious that by inserting Eq. (2) into (1), the constant τ_y cancels out from Navier-Stokes equation (at least for the high Reynolds flows or low yield stress fluids). Thus, provided a unified scaling procedure can be found for both models, proper initial and boundary conditions result in a same solution. It is common practice in literature to reduce data for H.-B. fluids with same scaling procedure used for P.-L. fluids (e.g. [5]). However, one can show that the results of such scaling do not exhibit similarity; the effects of the yield stress must be incorporated. Figure (1a) shows the application of P.-L. scaling on three different H.-B. fluids with same flow indices, but different yield stresses. The three curves do not collapse onto a single one. In the present work, we show that a correction must be applied to the wall shear stress, of the form

$$\tau_w = h\frac{\partial p}{\partial x} + \tau_y. \tag{3}$$

Use of the proposed classical scaling with the corrected wall shear stress (Eq. 3) results in an appropriate scaling that reduces all three fluids onto a single curve (Fig. 1b).
In the present work, both shear thinning and thickening fluids are considered (n ranging from 0.2 to 1.2). To solve the governing equations, the OpenFOAM code (2013) v.1.6-ext is used. The computations are carried out in a rectangular channel geometry with a long enough size to capture all representative frequencies (defined through an analysis of the two-point correlations). For all flow indices, the size of the channel is $(4\pi h,\ 2h,\ \pi h)$ (h = height).
The accuracy of the present DNS predictions was validated by comparison with Newtonian DNS data obtained through Spectral Methods. The non-Newtonian DNS data was validated against experimental data. Two grid resolutions were tested, a medium resolution with 256x192x192 points and a fine resolution with 512x384x384 points. To make the computations less consuming most computations were performed with the medium size grid. In the current study, Re_τ was limited to 395.
Figure 2a shows the mean velocity profiles for the viscous, buffer and fully turbulent regions. The expected linear behavior for the viscous layer ($u^+ = y^+$, u/u_τ = $(\rho u_\tau y)/\eta_w$)) and existence of a log-solution with same slope and different elevations for different flow indices is observed. The streamwise turbulent intensity increases as the flow index decreases. However, the other components of the Reynolds stress tensor show the opposite behavior. For the energy spectrum, the -5/3 exponent was retrieved in the inertial range, as expected.

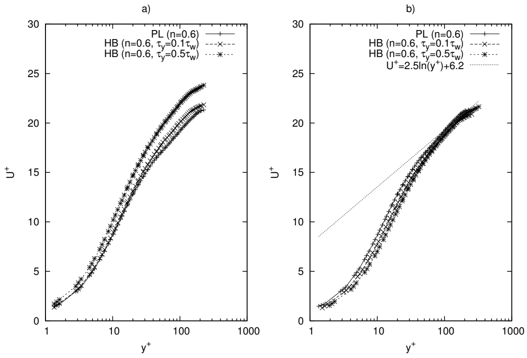
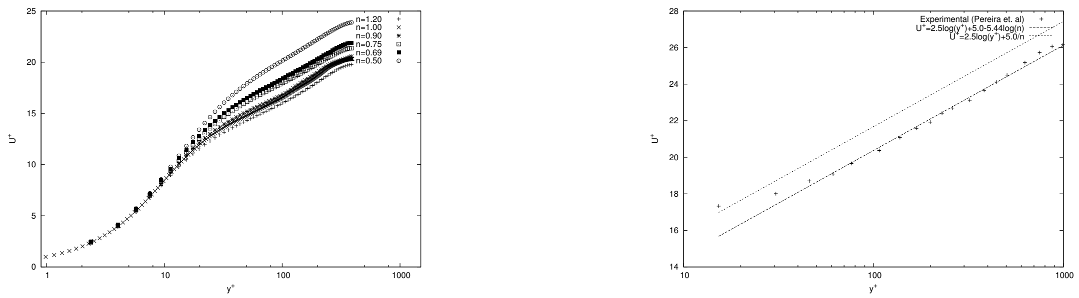


Figure 1: DNS results for turbulent flow of H.-B. fluids with same flow index and different yield stresses from [5]. a) Original scaling without considering yield stress, b) scaling considering the effect of yield stress in wall shear stress.



(a)

(b)

Figure 2: Mean axial velocity profile. a) DNS results, b) Comparison between the experimental results and the log-law, presented in the current work.

LAW OF THE WALL

One interesting application for the current DNS data set is the calibration of a generalized mean velocity profile for the turbulent flow of purely viscous non-Newtonian fluids. Mean velocity profiles of power-law fluids have been previously experimentally investigated by [2] and [3]. Recently, [4] showed analytically that for purely viscous non-Newtonian fluids, the indication is that the slope of the log-law is constant and equal to 2.5 (= κ^{-1}, the reciprocal of von Karman's constant). Some previous DNS results for flow index in the range $n = 0.5$ to 0.75, suggests that level of log-law must be $B(n) = \frac{5.0}{n}$. This elevation functional dependence works well for the limited range flow indices. However, a more detailed inspections (present work, n = 0.5, 0.69, 0.75, 0.9, 1, 1.2) shows that for low flow indices, specifically for n below 0.5, the agreement is not good.

In present work, the following functional behavior is observed to fit better the all available DNS data.

$$U^+ = 2.5 \ln\, y^+ + 5.0 - 5.44\, \ln\, n \tag{4}$$

The proposed universal profile reduces to the Newtonian case in the limit as n tends to unity. Also, it must be pointed out that this profile is valid for both power-law and H.-B. fluids. The applicability and accuracy of the presently proposed profile for higher Reynolds number flow is suggested in Fig. 2b.

References

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