THE EFFECT OF LARGE-SCALE INHOMOGENEITIES ON SMALL-SCALE STRUCTURE IN A TURBULENT FLOW

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INTRODUCTION

Kolmogorov's equation, which represents an energy balance for any particular scale of flow, provides a method for estimating the mean energy dissipation for HIT turbulence, viz. [3]

$$-\langle(\delta u)^3\rangle + 6\nu\frac{\mathrm{d}}{\mathrm{d}r}\langle(\delta u)^2\rangle = \frac{4}{5}\langle\epsilon\rangle r\ , \tag{1}$$

where $\delta u \equiv u(x+r) - u(x)$ is the longitudinal velocity increments (for the streamwise velocity component u), r is the distance between two points considered along the streamwise direction, x, and ν is the kinematic viscosity. Here, $\langle\epsilon\rangle$ is the mean dissipation rate of turbulent kinetic energy. However, the terms in Eq. (1) cannot be balanced for flows encountered in the laboratory conditions at moderate range of Reynolds numbers. The main reason for this imbalance is inhomogeneous large-scales. This led turbulence community to revisit Eq. (1) for turbulent flows, e.g. [1, 2]. There has been no previous study aimed at understanding the significance of inhomogeneous large-scales off the jet centreline. In this study, the effect of large-scale inhomogeneities on the smaller scales in a turbulent round jet is examined through a new generalized form of Eq. (1), viz.

$$\begin{aligned}&-\langle(\delta u)(\delta q)^2\rangle + 2\nu\frac{\mathrm{d}}{\mathrm{d}r}\langle(\delta q)^2\rangle - \frac{U}{r^2}\int_0^r s^2\frac{\partial}{\partial x}\langle(\delta q)^2\rangle\mathrm{d}s - 2\frac{\partial U}{\partial x}\frac{1}{r^2}\int_0^r s^2(\langle(\delta u)^2\rangle - \langle(\delta v)^2\rangle)\mathrm{d}s - 2\frac{\partial U}{\partial y}\\ &\frac{1}{r^2}\int_0^r s^2(\langle(\delta u)(\delta v)\rangle\mathrm{d}s - \frac{1}{r^2}\int_0^r s^2\frac{\partial}{\partial x}\langle(u+u^+)(\delta q)^2\rangle\mathrm{d}s - \frac{1}{r^2}\int_0^r s^2\frac{1}{y}\frac{\partial}{\partial y}(y\langle(v+v^+)(\delta q)^2\rangle)\mathrm{d}s = \frac{4}{3}\langle\epsilon\rangle r,\end{aligned} \tag{2}$$

where s is a dummy separation variable, y is the radial direction and the superscript $+$ refers to $(x+r)$. Here, $\langle(\delta q)^2\rangle$ ($\equiv \langle(\delta u)^2\rangle + 2\langle(\delta v)^2\rangle$) is defined as the total turbulent energy structure function. Dividing by $(4/3)\langle\epsilon\rangle r$, Eq. (2) can be rewritten as

$$A^* + B^* + D^* + P^* + S^* + I_u^* + I_v^* = C^*, \tag{3}$$

where A^* is the turbulent advection and B^* is the molecular diffusion term. Here, D^* is the inhomogeneous decay term along x and P^* is the energy production term. S^* is the mean shear term. I_u^* represents the effect of the inhomogeneity from the diffusion in the x direction while I_v^* represents the effect of the inhomogeneity from the diffusion in the y direction. Finally, C^* is the balance of all other terms, which is expected to be unity. Here, A^*, B^* and C^* are the classical terms in Eq. (1) while all other terms correspond the inhomogeneous large-scales.

Eq. (2) is of particular interest as it essentially represents an energy budget at different scales. It also provides a means for predicting the third-order structure function (the advection term), for a whole range of scales. As Eq. (2) is an exact relation between the second- and third-order structure functions, it can be useful for numerical studies, where structure functions are considered for sub-grid scale models. The validity of Eq. (2) is experimentally investigated here.

EXPERIMENTAL RESULTS

The experiments were carried out at the exit Reynolds number of $Re_D = 50,000$, where Re_D is calculated based on the jet exit mean velocity ($U_j = 10.65$ m/s) and the nozzle exit diameter $D = 0.0736$ m. The measurements were performed for $0 \leq y/D \leq 4.5$ and $10 \leq x/D \leq 20$. Measurements of the energy budget terms were obtained using a stationary cross-wire probe [4]. A novel flying hot-wire technique [5] was used to detect the influence of reverse flow. In Figure 1a, the normalized mean streamwise velocity is obtained by the stationary hot-wire and compared with the data obtained by the flying hot-wire at $x/D = 15$. The stationary hot-wire data are in good agreement with those from flying hot-wire for $0 \leq y/y_{0.5} \lesssim 1.25$. The departure between the stationary hot-wire and flying hot-wire data for $y/y_{0.5} \gtrsim 1.25$ is due the effect of reverse flow. In addition, a turbulent energy recognition algorithm (TERA) [6] was used to investigate the external intermittency in the current flow. Figure 1b provides the external intermittency factor, γ, across the jet centreline. For fully turbulent flow, γ is expected to be unity. The magnitude of γ is almost unchanged ($\gamma \simeq 1$) for $0 \leq y/y_{0.5} \lesssim 1$, and begins to decrease from around $y/y_{0.5} \simeq 1$. Therefore, Eq. (2) is experimentally investigated for $0 \leq y/y_{0.5} < 1$, where the flow is not affected by reverse flow or external intermittency. In order to illustrate the validity of Eq. (3), which is a

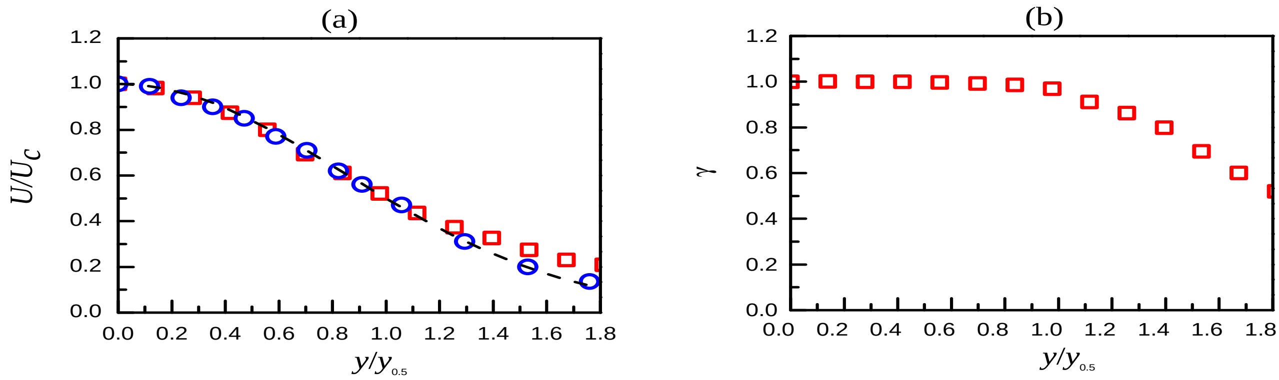

Figure 1: (a) Normalized mean streamwise velocity at $x/D = 15$. Squares are the mean velocity data taken by the stationary hot-wire. Circles are the mean velocity data taken by the flying hot-wire. Dashed line is $U/U_c(x) = \exp(-\ln 2((y/y_{0.5})^2)$. (b) Radial variation of the external intermittency factor γ at $x/D = 15$. Note $y_{0.5}$ is the jet half-radius and U_c is the velocity at the centreline.

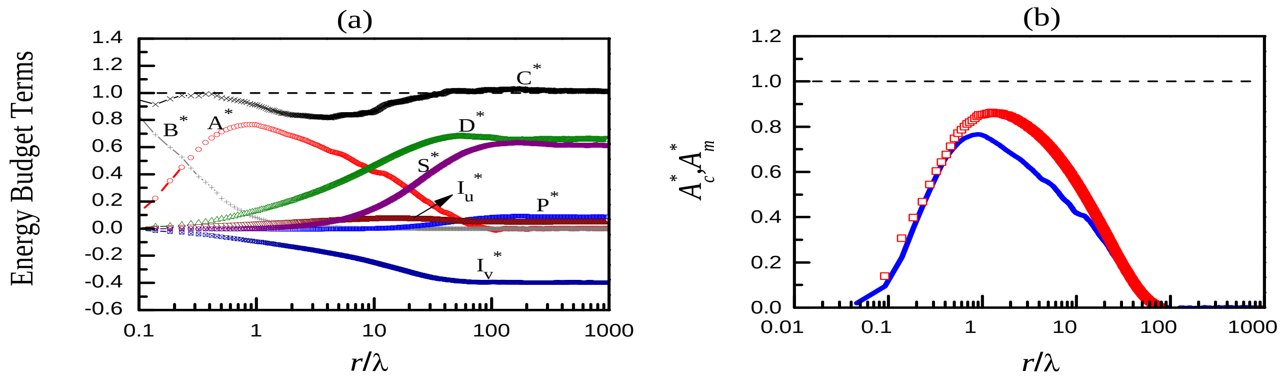

Figure 2: (a) Terms in (3) at $x/D = 15$ for $y/y_{0.5} = 0.42$. Red ◯ is A^*, gray + is B^*, green △ is D^*, blue □ is P^*, purple ⊗ is S^*, brown ▽ is I_u^*, royal ⊠ is I_v^* and black × is C^*. (b) Comparison between the measured value of the third-order structure function A_m^* (solid line) and that calculated in (3) denoted here as A_c^* (□).

normalised form of Eq. (2), the balance of all terms in this equation is investigated. Figure 2a shows different terms of Eq. (3) in terms of r/λ, where λ is the Taylor length-scale, for the radial position of $y/y_{0.5} = 0.42$ at $x/D = 15$. This figure shows that Eq. (3) is adequately satisfied by the experimental data (i.e., $A^* + B^* + D^* + P^* + S^* + I_u^* + I_v* = C^* \approx 1$). Eq. (2) can be used as a tool to calculate the third-order structure function. As such, A^* is calculated from this equation using the remaining terms (identified as A_c^*) and compared with the measured profile of A^* (denoted by A_m^*) in Figure 2b. A relatively good agreement is between A_m^* and A_c^*, which leads to the conclusion that third-order structure function (the advection term) can be estimated in jet flows when the second-order structure functions are known.

OUTLOOK

The final paper will focus on effects of large-scale inhomogeneities on different scales of flow at various radial locations across the jet centreline. In addition, more details about the implications of the new equation (2) will be provided. The similarity of the energy structure functions will also be investigated in the shear portion of the jet.

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